

Calculators and Mobile Phones are not allowed

Answer the following questions

Q1. (3 pts.) Use $(\epsilon-\delta)$ definition of the limit to prove that

$$\lim_{x \rightarrow -1} (3x + 9) = 6.$$

Q2. (3+3 pts.) Evaluate the following limits (if they exist)

$$(a) \lim_{x \rightarrow 1} (x-1)^2 \cos \frac{1}{x-1}, \quad (b) \lim_{x \rightarrow 2} \frac{1 - \sqrt{\frac{x}{2}}}{1 - \frac{x}{2}}.$$

Q3. (3 pts.) Find the value of the constant K so that the function f is continuous at $x = 2$, where

$$f(x) = \begin{cases} \frac{|x-2|}{x^2-4} & \text{if } x < 2 \\ \sqrt{x+2} + K & \text{if } x \geq 2. \end{cases}$$

Q4. (4 pts.) Find the horizontal and vertical asymptotes (if any) for the graph of the function f , where

$$f(x) = \frac{|x|(x+3)}{x^2-9}.$$

Q5. (3 pts.) If $\lim_{x \rightarrow 1} f(x) = 3$ and $\lim_{x \rightarrow 1} g(x) = 1$, find

$$\lim_{x \rightarrow 1} \frac{1 + [f(x)]^3}{\sqrt{g(x)} - 4}.$$

Q6. (3 pts.) Use the Intermediate Value Theorem to show that the equation

$$\cos x + 2x - 2 = 0$$

has at least one real root.

Q7. (3 pts.) Find the x -coordinates of the points at which the function f is discontinuous, where

$$f(x) = \frac{x^3 - 1}{x^2 + 2x - 3}$$

Classify the types of discontinuity of f as removable, jump, or infinite.

(Good Luck)